RAYNHAM PRIMARY SCHOOL

Teaching calculation strategies that lead onto written methods of calculation

Raynham Primary School

Autumn 2022

PROUD TO BE PART OF Children First Academy Trust

Introduction: The aim is for all children to be confident in at least one written method for each of the four operations which is **reliable** and **efficient**.

- There is progression in calculation from Foundation Stage to Y6.

- In order for progression to run smoothly, vocabulary, mental calculation strategies and rapid recall facts should be introduced at the appropriate stage, used in context, and reinforced regularly. Interpretation of written signs /symbols also needed to be taught.

- The different approaches to mental and written calculation should be made explicit.

- Numerical examples have been included to illustrate progression; however, the Framework for Teaching Mathematics, including the Pitch and Expectations document should be referred to for appropriate year groups and levels of attainment.

- All 4 operations should be reinforced in the 'Using & Applying' strand by making links to maths in real life contexts.

- Mental warm ups could include daily counting practice in all year groups.

Addition

By the end of Foundation Stage

Understand concept and vocabulary of plus, add, more total, sum and altogether. The vocabulary should be taught through practical activities in meaningful contexts up to 10.

- Songs – 1,2,3,4,5 Once I caught a fish alive, (See Learn to CROW with pictures)
- Count on, altogether, one more etc
- Matching numbers to objects
- Counting on fingers in a consistent way
- Counting forwards in 1's, 2's and 10's, up to 100, 20, 100 respectively (Ext 5's) on a number line and number square .

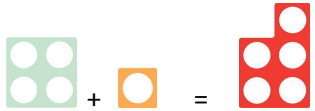
0 2 4 6 8

- Objects and Marks – 1 more / 2 more

There are 3 cars in the garage. 1 more came along

3 + 1

Use Numicon/ or other concrete apparatus



4 + 1 = 5

Terry has 3 apples and Tony has 2 apples. How many altogether?

+

- Begin to work on number bonds to 10. Ext to 20

By the end of Y1

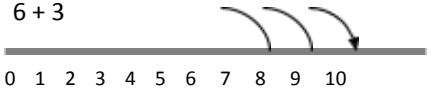
Through practical activities in meaningful contexts and informal written methods

- Develop partitioning/bonds to 20 and within 20 (separate a group of objects into 2 groups) e.g. 8 is 5 and 3. Reinforce with Numicon.
- Use of number sentences and mathematical language - count on / altogether / one more/ two more

- Use of number lines and 100 square within 10 / Ext 50

Drawing jumps on prepared lines

6 + 3



0 1 2 3 4 5 6 7 8 9 10

Relationships/Related Facts including symbols + and = various positions in number sentence:

□ = 5 + 4

5 + 4 = □

5 + □ = 9

□ + 4 = 9

□ = 5 + 4

□ + □ = 9

Partitioning 14 = 10 + 4 (Numicon)

- Money and addition up to 50p

(Blu tack Numicon to coins)

By the end of Y2

Through practical activities and meaningful contexts and informal written methods

- Fluent recall of bonds to 20 and within 20
- Derive fact families up to 100

13 + 7 = 20

7 + 13 = 20 (and link to subtraction)

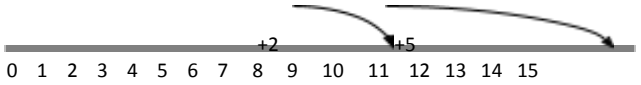
20 = 7 + 13

20 – 7 =13

20 - 13 = 7

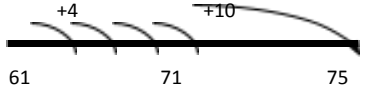
- Constructing own lines- partitioning the smallest number

8 + 7 = 15



(Allow children to make choices when partitioning the smaller number, not all children will need to partition the smaller number, some will count in 1s.)

- Partitioning into tens and ones and using number lines e.g. 61+14



(Teach children to add ones first to ensure) consistency when moving onto column method

- Use Numicon to help partitioning and re-combining

61 + 14 = 60 + 1

10 + 4

70 + 5

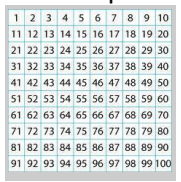
61 + 14 =

1 + 4 = 5

60 + 10 =70

70 + 5 = 75

- Addition of money up to £1.00 – using coins

● Money – Use concrete objects to help recognition value of coins and simple addition e.g. 1p + 2p **Outdoor curriculum:** ○ Role-playing number songs ○ Matching numbers to objects: Number in wheel barrows, adding correct number of blocks / bean bags. Numbers in hoops, throwing correct number of beanbags into hoops. ○ Counting forward in 1's: Skipping, Jumping on numbers, Shooting hoops ○ Objects and Marks Chalk numbers ○ Number bonds: Objects in hoops Bowling - Working out how many pins has been rolled over during each frame of 2 throws each.	● Count in 2's, 5's and 10's <u>Extension</u> 5 + 10 ● Adding multiples of 10 5 + 20 3 + 30 ● Adding three numbers 2 + 8 + 4 = 14 or 1 + □ + 5 = 9 ● Extend to questions such as 14 + 5 = 10 + □ **National Curriculum requirements:** Add 1 digit and 2 digit numbers to 20, including 0.	● <u>Count on in 2's, 5's and 10's from different numbers</u> e.g. 3, 9, 16, within 100. ● Visualisation of the 100 square  ● Column addition – no carrying **National Curriculum requirements:** *(using concrete objects, pictorial representations and mentally)* Add 2 digit numbers and ones. Add 2 digit number and tens. Add two 2 digit numbers. Add three 1 digit numbers.

To add successfully, children need to be able to:

- Recall all addition pairs to 9 + 9 and know number bonds to and within 10/20/100
- Add mentally a series of one-digit numbers, such as 5 + 8 + 4
- Add multiples of 10 (such as 60 + 70) and 100 (such as 600 + 700) using the related addition fact, e.g. 6 + 7, and their knowledge of place value
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways. e.g. 52 = 50 + 2, 40 + 12, 30 + 22

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for addition.

In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations using H T 0.t h to reinforce the place value of each digit in the sum.

National Curriculum requirements:
Year 3: Add numbers with up to 3 digits, using the formal written method of columnar addition.

Year 4: Add numbers with up to 4 digits, using the formal written method of columnar addition.

Year 5: Add whole numbers with more than 4 digits, using the formal written method of columnar addition.

Year 6: Add whole numbers with more than 4 digits, using the formal written method of columnar addition.

Progression examples

Y3

To add successfully, children need to be able to:

1. No 'carrying'

T0	HT0
23	315
+ 42	+ 624
65	939
	Ext

2. Extra digit in answer

T0	HT0
94	561
+ 73	+ 718
167	1279
	Ext

3. Carrying 0 to T

T0	HT0
1	1
47	237
+25	+ 516
72	753
	Ext

NB. Emphasise value of digit e.g. 2 tens and 3 ones
Continue to use number lines and partitioning method.
Use resources such as base 10 to model/aid. Write calculations horizontally.
Emphasise as a school, we will 'carry' over the digits at the top for all column addition.
Carry digits are recoded above the line using the words 'carry 10' or carry '100' and not 'carry 1'.

Y4

4. Carrying T to H.

HT0
1
371
+ 485
856

5. Carrying 0 to T and T to H

HT0
1 1
376
+ 485
861

6. Different number of Digits

ThHT0
1 1
2388
+ 1124
3512

Money and Decimals

A. £3.25 + £1.53 = £4.78

B.

	1
£4.21	
+ £3.87	
£8.08	

£1.85 + 48p (written horizontally)

C.

	1 1
£1.85	
+ £0.48	
£2.33	

Y5

Addition using Th, H, T, 0 including more than two numbers

D.

	2 1 1
1463	
+ 921	
759	
3143	
211	

Addition of decimals T0.t h

E. 14.1 + 3.89

14.10
+ 3.89
17.99

Real life problems involving money or measures.

Y6

Reinforce and securing all of the above with increased emphasis on using and applying in preparation for secondary transfer.

Subtraction

By the end of Foundation Stage

Understand concept and vocabulary of -, difference, subtraction, less, minus, take away through practical activities in meaningful contexts within 5 and 10

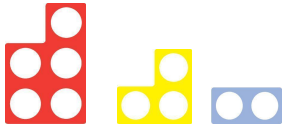
- Songs such as 5 little ducks went swimming one day, 10 in a bed, 10 fat sausages…..

(using images such as pegs)
e.g. 1 less than 5 (cover 1 peg up)

- 1 less , 2 less
- Counting on from a smaller number
- Counting back through practical activities in meaningful contexts
- Objects/ Marks
  We made 6 cakes. We ate 2 of them.
  How many cakes are left?

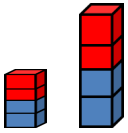
Link to number line 6 – 2 =

0 1 2 3 4 5 6



5 - 3 = 2

Use Numicon/ Or other concrete objects



The difference between 2 and 4

Ext: Begin to understand inverse of number bonds to 10.
e.g. 10 – 8 = 2

Outdoor curriculum:

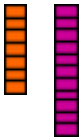
- Role-playing number songs

Pegs and numerals on a washing line.

- 1 less / 2 less

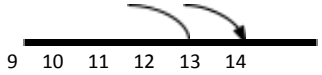
By the end of Y1

Through practical and meaningful contexts



- Find the difference within 20,
  e.g. The difference between 7 and 11 using towers/Numicon/Cuisenaire rods
- Drawing jumps on using horizontal/vertical number lines
- Find difference by counting up (only when the difference is small)

14 – 12 = 2



- Subtract multiples of 10 e.g. 50 – 20 = 30
- Record using – and =

Relationships/ Related Facts

5 - 2 = □ □ = 5 - 2
5 - □ = 3 3 = □ - 2
□ - 2 = 3 3 = 5 - □
□ - □ = 3 3 = □ - □

Use of balance

5 – 2 = □

- Continued use of Numicon
  12 – 8

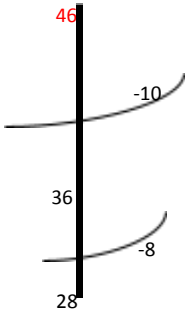
- Consolidate inverse of number bonds to 20
  e.g. 10 – 3 = 7
  10 – 7 = 3
- Consolidate inverse of number bonds within 20
  e.g. if 6 + 2 = 8
  8 – 2 = 6
  8 – 6 = 2
- Add and subtract one digit and two digit numbers to 20 including 0.

National Curriculum requirements:
Subtract 1 digit and 2 digit numbers up to 20, including 0.
Represent and use number bonds and related subtraction facts.

By the end of Y2

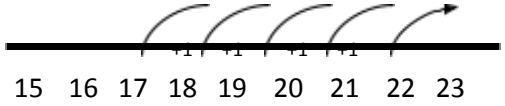
Through practical and meaningful contexts

- Counting back by partitioning second number.
  Eg 46 - 18
  46 – 10 - 8



- Find the difference (subtraction) by counting on and add only when the difference is small.

example 1 23 – 18 = 5



Use Numicon to support

- Inverse bonds fluently to 20 and within 20
  e.g. 20 – 18 = 2
  20 – 2 = 18

  16 minus 4 =12
  16 subtract 12 = 4

- Derive and use related facts up to 100
  e.g. 10 – 7 = 3 so 100 – 70 = 30

- Column subtraction – no exchange

National Curriculum requirements:

Hoops & Bean bags ○ Problem solving: Tricycles: There are 5 tricycles. 2 are being use. How many are left? 		(using concrete objects, pictorial representations and mentally) Subtract 2 digit numbers and ones. Subtract 2 digit number and tens. Subtract two 2 digit numbers. Subtract three 1 digit numbers.

To subtract successfully, children need to be able to:

- Recall subtraction facts to 20 and within 20
- Subtract multiples of 10 (such as 160 – 70) using the related subtraction fact 16 – 7 and their knowledge of place value
- Partition two-digit and three-digit numbers into multiples of hundreds, tens and ones/units in different ways (e.g. partition 74 into 70 + 4 or 60 + 14).

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for subtraction.

In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations using H T 0.t h to reinforce the place value of each digit in the calculation.

National Curriculum requirements:
Year 3: Subtract numbers with up to 3 digits using the formal written method of columnar subtraction.

Year 4: Subtract numbers up to 4 digits using the formal written method of columnar subtraction.

Year 5: Subtract numbers with more than 4 digits.

Year 6: Subtract numbers with more than 4 digits.

Progression examples

Y3

N.B. Emphasise value of digit, e.g. 4 tens subtract 2 tens = 2 tens

1. No adjustment

T0	HT0	
47	864	
- 23	- 621	
24	243 Ext	

2. Adjustment / Exchange T and 0

T 0	
$^{4}\cancel{5}^{1}1$	to aid use base
- 3 6	10 to help
1 5	

- Consolidation of using number lines, counting forwards and backwards within 100. .
- Use the correct language for subtraction eg: exchange rather than borrow

	Y4	3. Adjustment H to T H T 0 $^{3}\cancel{4}\,^{1}3\,7$ - 1 8 2 2 5 5 4. Adjustment H to T and T to 0 H T 0 $^{3}\cancel{4}\,^{12}\cancel{3}\,^{1}2$ - 1 8 7 2 4 5 5. Noughts H T 0 $4\,\cancel{6}\,^{5}\cancel{7}\,^{1}0$ -1 4 2 3 2 8 H T 0 $^{5}\cancel{6}\,^{9}\cancel{0}\,^{1}4$ - 3 4 7 2 5 7 Check using inverse Extend to subtracting numbers up to 4 digits **A.** £4.35 - £1.23 £3.12 **B.** £5.$^{3}\cancel{4}\,^{1}$5 - £2. 2 9 £3. 1 6 **C.** £$^{2}\cancel{3}\,^{1}$.59 - £ 1.7 3 £ 1. 86
	Y5	**Subtraction involving Th, H, T 0 and use of number lines (including decimals also)** **A.** £$^{4}\cancel{5}.^{12}3\,^{1}\cancel{5}$ - £ 2. 8 7 £ 2. 4 8 **B.** £ 3.$^{7}8\,^{1}\cancel{0}$ - £ 1. 3 9 £ 2. 4 1 **C.** £$^{3}4.\,^{1}\cancel{0}\,^{1}\cancel{0}$ - £ 1. 3 5 **£** 2. 6 5 **D.** £ $^{7}\cancel{8}.^{1}$08 - £ 2. 71 £ 5. 37 Real life problems involving money or measures.
	Y6	**Reinforce and securing all of the above.**

Multiplication

By the end of Foundation Stage

Understand concept and vocabulary of multiplication (double, lots of) through practical activities in meaningful contexts

- Counting in 2's and 10's (Extend to 5's)
- Doubles up to 5 + 5
- Double objects practically:

\+ =

Outdoor Curriculum:
Practical problems –

- There are 2 children and they each need 4 balls. How many do they need to get altogether?

\+ =

- How many wheels do 2 tricycles have?

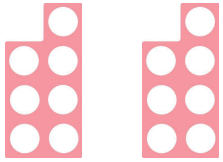
\+ =

- Board games: Doubling Ladybirds Dots

By the end of Y1

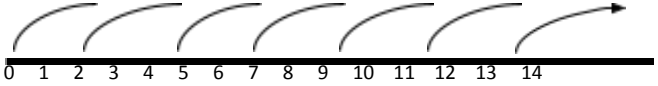
Through practical activities and meaningful contexts

- Doubles up to 10 + 10 (Use Numicon)

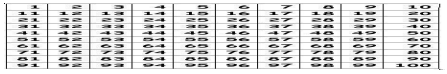


7 + 7 =14

- Counting in 2's, 5's to 50 and 10's to 100
- Continue to use Number Lines



- Use of "100 Square" up to 20 to count in 2's, 5's and 10's up to 50/100



- One step problems e.g: how many wheels do we need to make three Noddy cars? 5 + 5 + 5 = 15

- Counting multiples of coins: 2p, 5p, 10p

2p + 2p + 2p

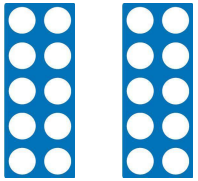
National Curriculum requirements: Solve one step problems involving multiplication, by calculating the answer

By the end of Y2

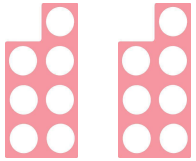
Through practical activities and meaningful contexts

- Doubles numbers up to 20+20 (by partitioning and recombining)

17 + 17 (Use Numicon)



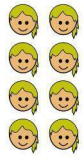
10 + 10



7 + 7

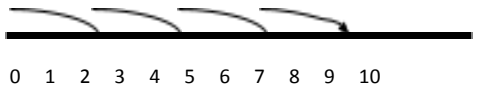
(Leading to abstract thinking)

- Understanding multiplication as repeated addition/ groups/ lots.
- Reading arrays - 2x4 (2, 4 times)

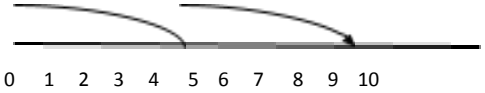


- Remind children of repeated addition learnt in previous years.

2 + 2 + 2 + 2 (4 groups of 2)



4 + 4 (2 groups of 4)



- Know the times tables and division facts for 2, 5 and 10

using concrete objects, pictorial representations and arrays with the support of the teacher.

National Curriculum requirements: Solve problems involving multiplication using materials, arrays, mental methods and multiplication facts.

To multiply successfully, children need to be able to:

- Recall multiplication facts to 12 x 12
- Partition numbers into multiples of hundreds, tens and ones/units
- Work out products such as 70 x 5, 70 x 50, 700 x 5 or 700 x 50 using the related fact 7 x 5 and their knowledge of place value
- Add two or more single-digit numbers mentally
- Add multiples of 10 (such as 60 + 70) or of 100 (such as 600 + 700) using the related addition fact, 6 + 7, and their knowledge of place value
- Add combinations of whole numbers using the column method

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for multiplication.

In Y3 and Y4 (and beyond where appropriate) the children should record their written calculations' using Th H T 0.t h to reinforce the place value of each digit in the calculation.

National Curriculum requirements:
Year 3: Multiply 2 digits by 1 digit, using mental and progressing to formal written methods.

Year 4: Multiply 2 digits by 1 digit using formal written layout. Multiply 3 digits by 1 digit using formal written layout.

Year 5: Multiply numbers up to 4 digits by a 1 digit number using the formal written method of short multiplication. Multiply numbers up to 4 digits by a 2 digit number using the formal written method of long multiplication. Multiple whole numbers and those involving decimals by 10, 100, 1000.

Year 6: Multiply up to 4 digits by 2 digits using the formal written method of long multiplication.

Progression examples

Y3

- Know Times Tables and division facts for 2, 3, 4, 8
- Continue to use arrays and number lines/Cuisenaire rods/Numicon for x 2, 3, 4, 8
- Introduce grid method

X	10	4
5	50	20 = 70

(Link to image of partitioning array/use place value arrow cards)

- No carrying

TO
32
X 3
96

Extra digit

HTO
51
x 2
102

Y4

- Know Times Tables and division facts up to 12x12
- Short Multiplication

Carrying

	5
	3 8
X	7
	2 6 6

Noughts

	202
x	4
	808

Ext

	5 □
x	4
	2 □ 2

Ext

	234
	x 63
	702
+	14040
	14742

Y5

Use Grid Method to introduce TO x TO then introduce Long Multiplication.
Introduce Long Multiplication

	H	T	O
	1	2	
	2	2	5
x			5
	1	1	2 5

		4	0 7
x		7	8
	3	2	5 6
	8	4	9 0
	3	1	7 4 6

Extend to ThHT0 x T0

Y6

Introduce Long Multiplication

	9	8	6 x
		6	9
	8	8	7 4
+	5	9	1 6 0
	6	8	0 3 4

Multiply numbers by 10,100, 1000 giving answers up to 3 decimal places.		[1] ● Introduce multiplying decimals using column 0.t x 0 and U.t h x 0 1.3 x 4 =5.2 3.65 x 5 = 18.25

Division

By the end of Foundation Stage

- Understand concept and vocabulary of division (sharing, shares, equal groups, 1 whole/half) through practical activities in meaningful context

Sharing 6 cakes between 2 people

Share a bag of 10 sweets between 2 children – one for you, one for me

- Grouping objects equally

How many pairs of socks are there in the "launderette"?

10 grouped into 2s How many groups?

- Introduce halving even numbers up to 10

Using multilink, counters etc.

Half of 4

- Halving a whole object.

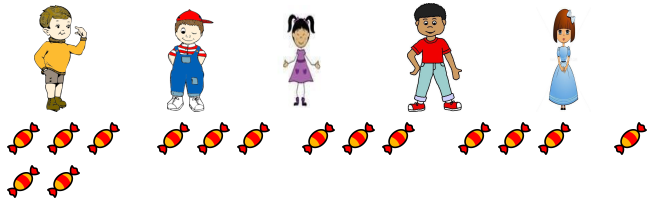
By the end of Y1

Through practical activities in meaningful contexts

- Division as Sharing

Share equally

Share a bag of 15 sweets between 5 children – one for you, one for you, one for you, one for you, one for me

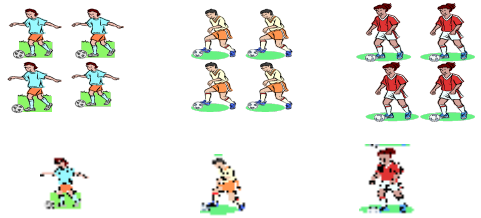


- Introduce number sentences using ÷ sign

- Division as Grouping, 2's, 5's and 10's

A bag of sweets, how many children can have 2 sweets each? - Put into groups of 2

15 children get into teams of 5 to play a game. How many teams are there?



- Consolidate halving even numbers up to 10 and link to inverse of multiplication (Ext to 20) Continue to use multilink.

Understanding 8 ÷ 2 as half of 8

- Use of Numicon and Numicon Number Lines

10

How many 2's in 10 and 5's in 20?

By the end of Y2

Through practical activities in meaningful contexts

- Know by heart, half all numbers up to 20

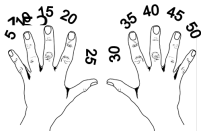
- Halving multiples of 10 up to 100

- Recognise relationship between x and ÷

- Use Numicon and hands

to help with how many groups of 5 in

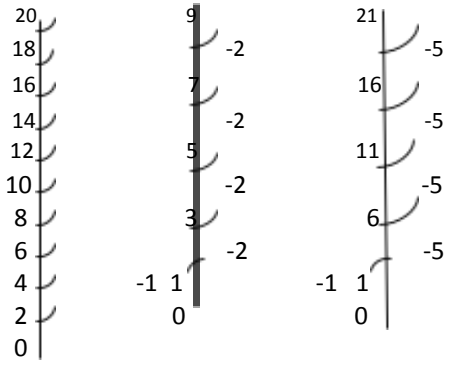
How many 5's have been counted?



Using number lines to answer questions such as

20 ÷ 2 linking to how many 2's in 20?

Include calculations with remainders 9 ÷ 2 = 4 r1



A 9÷2=4r1 21 ÷ 5 = 4 r 1

Use Cuisenaire rods and number track.

Know related division facts for 2 x 5 x 10 x tables

Record using ÷ and = signs

Outdoor curriculum: Practical problems – There are 2 children and 6 balls. To share them equally, how many does each get? 	**National Curriculum requirements:** Solve one step problems involving division, by calculating the answer by using concrete objects, pictorial representations and arrays with the support of the teacher.	**National Curriculum requirements:** Solve problems involving division using materials, mental methods and division facts.

To divide successfully in their head, children need to be able to:

- Recall division facts of times tables up to 12
- Understand and use the vocabulary of division – for example in 18 ÷ 3 = 6, the 18 is the dividend, the 3 is the divisor and the 6 is the quotient
- Partition two-digit and three-digit numbers into multiples of 100, 10 and 1 in different ways,
- Recall multiplication facts of one-digit numbers and divide multiples of 10 or 100 by a single digit number using their knowledge of division facts and place value
- Know how to find a remainder working mentally, for example, find the remainder when 48 is divided by 5
- Understand and use multiplication and division as inverse operations.

Note: It is important that children's mental methods of calculation are practised and secured alongside their learning and use of an efficient written method for division

To carry out written methods of division successful, children also need to be able to:

- Be able to add multiples of 2 digit numbers.
  eg 23 + 23 + 23. See example for Y6
- Estimate how many times one number divides into another – for example, how many sixes there are in 47, or how many 23s there are in 92
- Multiply a two-digit teen number by a single-digit number mentally
- Subtract numbers using the column method.

National Curriculum requirements:
Year 3: Division questions based on multiplication tables they know.
Divide 2 digits by 1 digit, progressing to formal written methods.

Progression examples

Y3

Know division facts for 2, 3, 4, 8 times tables
Introduce chunking before short division

```
   13
5) 65
 - 50   5 x 10
   15
 - 15   5 x 3
   0    13
```

No remainder, no carrying

```
   23
3 ) 69
```

Remainder, no carrying

```
   22 r 2
3 ) 68
```

Continued use of Cuisenaire and number lines to model.

Y4

Know all division facts for all times tables

No remainder, carrying

```
   1 5
3 ) 4 ¹5
```

Remainder, carrying

```
   1 4 1  r1
3 ) 4 ¹2 4
```

Placing of quotient (answer)

```
   4 1
7 ) 2 ²8 7
```

Noughts in quotient

```
   2 0 4
4 ) 8 1 ¹6
```

Extend to HT0 ÷ T0

Y5

Consolidate short division

Continue to practise chunking

2.

```
   24
4 ) 96
 -80   4 x 20
  16
 - 16   4 x 4
   0    24
```

Answer = 24

3. **Extend to**

```
   61
13 ) 793
 - 78 |  13 x 6
   1 ↓
 - 13   13 x 1
   13
   13
    0   61
```

Answer = 61

Extend to include remainders appropriate for the context (using whole numbers, simplified fractions or decimals)

Y6

Consolidate chunking

The National Curriculum statutory requirements for Year 3 and the use of written methods are not clear therefore our guidance for Year 3 has been based on the skills required to access Year 4 statutory requirements.

Year 4:

Divide 2 digits by 1 digit and 3 digits by 1 digit becoming fluent with formal written method of short division with exact answers and progressing to remainders.

The National Curriculum statutory requirements for Year 4 and the use of written methods are not clear therefore our guidance for Year 4 has been based on the skills required to access Year 5 statutory requirements.

Year 5: Divide 2 digits by 1 digit.

Divide 3 digits by 1 digit.

Divide 4 digits by 1 digit.

Children interpret the remainders appropriately for the context.

e.g. as fractions, decimals or by rounding

98÷4 = 98/4 = 24r2 = 24 ½ = 24.5 rounded to 25

Divide whole numbers and those involving decimals by 10, 100, and 1000.

Year 6: Divide numbers up to 4 digits by a 2 digit number using the formal written method of short division where appropriate.

Divide up to 4 digits by a 2 digits whole number using the formal written method of long division.

Introduce division by 2 digit numbers using long division method

Extend to dividing decimals

Children should be able to show a remainder as a whole number, simplified fraction or decimal.

use of calculator

98 ÷ 7 becomes

1 4
7 | 9 ^{2}8

Answer: 14

432 ÷ 5 becomes

8 6 r 2
5 | 4 3 ^{3}2

Answer: 86 remainder 2

496 ÷ 11 becomes

4 5 r 1
1 1 | 4 9 ^{5}6

Answer: $45 \frac{1}{11}$

1 5 | 4 3 2
3 0 0
1 3 2
1 2 0
1 2

Answer: 28 remainder 12

1 5 | 4 3 2
3 0 0 (15×20)
1 3 2
1 2 0 (15×8)
1 2

$\frac{12}{15} = \frac{4}{5}$

Answer: $28 \frac{4}{5}$

1 5 | 4 3 2 · 0
3 0
1 3 2
1 2 0
1 2 0
1 2 0
0

Answer: 28·8